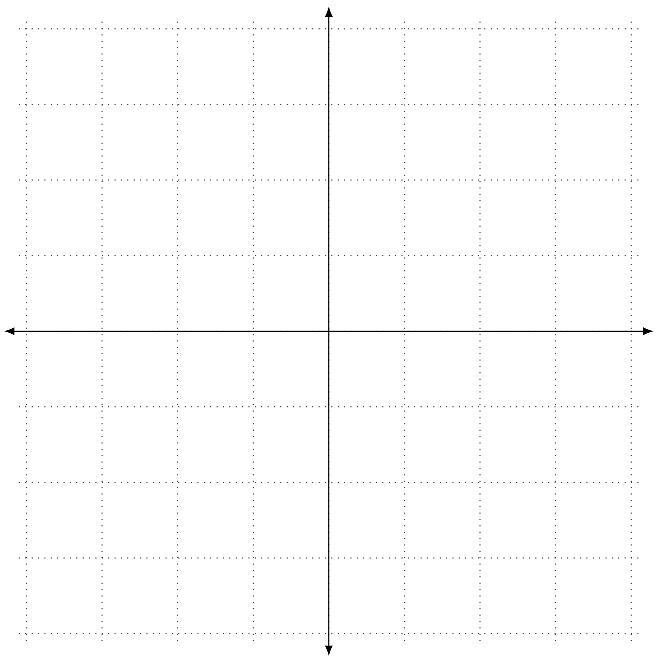
27

27.1 Draw $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and *all* vectors orthogonal to it. Call this set A.

27.2 If $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\vec{x}$ is orthogonal to $\vec{u}$, what is $\vec{x} \cdot \vec{u}$?

27.3 Expand the dot product $\vec{u} \cdot \vec{x}$ to get an equation for A.

27.4 If possible, express A as a span.



Normal Vector

DEF

A ***normal vector*** to a line (or plane or hyperplane) is a non-zero vector that is orthogonal to all direction vectors for the line (or plane or hyperplane).

28 Let $\vec{d} = \begin{bmatrix}1\\2\end{bmatrix}$ and $\vec{p} = \begin{bmatrix}1\\-1\end{bmatrix}$, and define the lines

$$\ell_1 = \operatorname{span}\{\vec{d}\} \qquad \text{and} \qquad \ell_2 = \operatorname{span}\{\vec{d}\} + \{\vec{p}\}.$$

28.1 Find a vector $\vec{n}$ that is a normal vector for both ℓ_1 and ℓ_2.

28.2 Let $\vec{v} \in \ell_1$ and $\vec{u} \in \ell_2$. What is $\vec{n} \cdot \vec{v}$? What about $\vec{n} \cdot (\vec{u} - \vec{p})$? Explain using a picture.

28.3 A line is expressed in *normal form* if it is represented by an equation of the form $\vec{n} \cdot (\vec{x} - \vec{q}) = 0$ for some $\vec{n}$ and $\vec{q}$. Express ℓ_1 and ℓ_2 in normal form.

28.4 Some textbooks would claim that ℓ_2 could be expressed in normal form as $\begin{bmatrix}2\\-1\end{bmatrix} \cdot \vec{x} = 3$. How does this relate to the $\vec{n} \cdot (\vec{x} - \vec{p}) = 0$ normal form? Where does the 3 come from?

29

Let $\vec{n} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

29.1 Use set-builder notation to write down the set, X, of all vectors orthogonal to $\vec{n}$. Describe this set geometrically.

29.2 Describe X using an equation.

29.3 Describe X as a span.